IFIC/01–39
FTUV/01–0731

On the massless contributions to the vacuum polarization of heavy quarks

J. Portolés and P. D. Ruiz-Femenía

Departament de Física Teòrica, IFIC, Universitat de València - CSIC
Apt. Correus 22085, E-46071 València, Spain

Abstract

In Ref. [1] Groote and Pivovarov have given notice of a possible fault in the use of sum rules involving two–point correlation functions to extract information on heavy quark parameters, due to the presence of massless contributions that invalidate the construction of moments of the spectral densities. Here we show how to circumvent this problem through a new definition of the moments, providing an infrared safe and consistent procedure.

PACS : 11.55.Hx, 11.55.Fv, 13.65.+i, 12.38.Bx
Keywords : Heavy quark sum rules, singularities of perturbative amplitudes.

1 Introduction

The vacuum polarization function suitable for extracting fundamental information of heavy quark-antiquark systems is built from the vector current $j^\mu(x) = \overline{Q}(x)\,\gamma^\mu\,Q(x)$ of the heavy quark Q of mass M :

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\,\langle 0|T\, j_\mu(x)\, j_\nu^\dagger(0)\,|0\rangle \;=\; \left(-g_{\mu\nu}q^2 + q_\mu q_\nu\right)\Pi(q^2)\,. \tag{1}$$

As it is well known two–point functions are analytic except for singularities at simple poles or branch cuts, the latter being originated by normal thresholds of production of internal on–shell states. Implicitly assuming that the absorptive part of $\Pi(q^2)$ starts at the massive two–particle threshold $q^2 = 4M^2$, vanishing below this point, the correlator satisfies the once–subtracted dispersion relation [2] [1] :

$$\widehat{\Pi}(q^2) \;\doteq\; \Pi(q^2) \;-\; \Pi(0) \;=\; \frac{q^2}{\pi}\int_{4M^2}^{\infty}\frac{ds}{s}\;\frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon}\quad . \tag{2}$$

This dispersion relation has been extensively used to determine heavy quark parameters within the method of sum rules because it allows to relate experimental input, on the right–hand side, with theoretical perturbative evaluations on the left–hand side [3]. Indeed $\mathrm{Im}\,\Pi(q^2)$ refers to the total cross section of heavy quark production $\sigma(e^+e^-\to Q\overline{Q})$. In practice, the spectral density $\mathrm{Im}\,\Pi(s)$ is poorly known experimentally at very high energies and, in addition, we are interested in the very low energy domain because it is more sensitive to the heavy quark mass. Therefore one uses derivatives of the vacuum polarization at the origin, called moments, to be responsive to the threshold region :

$$\mathcal{M}_n = \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)|_{q^2=0}\quad . \tag{3}$$

Until present the evaluation of the perturbative two–point correlation function $\Pi(q^2)$ has only been carried out completely, with massive quarks, up to $\mathcal{O}(\alpha_s^2)$ [4] and the procedure above has been termed consistent and effective in its task because the first branch point is set at the massive two–particle threshold. However in Ref. [1] Groote and Pivovarov have pointed out that at $\mathcal{O}(\alpha_s^3)$ there is a contribution to the correlator which contains a three–gluon massless intermediate state (see Fig. 1(a)). Its absorptive part starts at zero energy and, therefore, Eq. (2) is no longer correct. Moreover those authors have also warned about the fact that, at this perturbative order, the massless intermediate state invalidates the definition of the moments $\mathcal{M}_n$ for $n \geq 4$ because they become singular. In Ref. [5] an infrared safe redefinition of the moments, to cure the latter problem, has been provided; it consists in evaluating the moments at an Euclidean point $q^2 = -s_E$, $s_E > 0$, thus avoiding the singular behaviour. Nevertheless the fault in Eq. (2) due to the massless threshold still

[1]Sometimes de Adler function defined as $\partial\Pi(q^2)/\partial\ln q^2$, to get rid of the subtraction constant, is used.

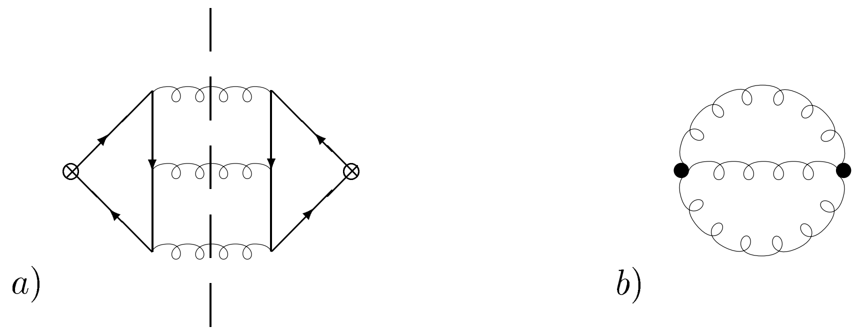


Figure 1: *(a) $\mathcal{O}(\alpha_s^3)$ diagram contributing to the vacuum polarization function of the heavy quark current (the vertical dashed line indicates the massless cut). (b) "Effective" diagram obtained by integrating out the fermion loops. It also has the topological structure of the "reduced" diagram that determines the massless cut singularity.*

represents a problem because even if, as we will justify later on, we substitute this dispersion relation by

$$\widehat{\Pi}(q^2) \;=\; \frac{q^2}{\pi}\int_{4M^2}^{\infty} \frac{ds}{s}\,\frac{\mathrm{Im}\,\Pi_{Q\overline{Q}}(s)}{s-q^2-i\epsilon} \;+\; \frac{q^2}{\pi}\int_{0}^{\infty} \frac{ds}{s}\,\frac{\mathrm{Im}\,\Pi_{3g}(s)}{s-q^2-i\epsilon} \quad , \tag{4}$$

(where the notation is self-explicative), the spectral function $\mathrm{Im}\,\Pi_{3g}(s)$ associated to the cut in Fig. 1(a) would hardly be implemented phenomenologically as gluons hadronize to both heavy and light quark pairs. Perturbatively the three–gluon cut would contribute to $Q\overline{Q}$ production, i.e. to $\mathrm{Im}\,\Pi_{Q\overline{Q}}(q^2)$, but at higher order in α_s. Therefore if we attach to an $\mathcal{O}(\alpha_s^3)$ sum rule analysis, that contribution should be extracted from the perturbative $\Pi(q^2)$ evaluation. In this note we provide a bypass to recover the balance between the right-hand and left-hand parts of Eq. (4).

2 Moments and the massless cut

The perturbative contribution given by the diagram in Fig. 1(a) has been calculated at small q^2 ($q^2 \ll M^2$) in Ref. [1]. In this limit the quark triangle loop can be integrated out and it ends up in the diagram in Fig. 1(b) generated by an induced effective current describing the interaction of the vector current with three gluons [2],

$$J^\mu \;=\; -\frac{\pi}{180M^4}\left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}}\left(5\,\partial_\nu \mathcal{O}_1^{\mu\nu} \;+\; 14\,\partial_\nu \mathcal{O}_2^{\mu\nu}\right)\,, \tag{5}$$

[2]The permutations of the three gluons in Fig. 1(a) are already included in the definition of the effective current.

with

$$
\begin{aligned}
\mathcal{O}_1^{\mu\nu} &= d_{abc}\, G_a^{\mu\nu} G_b^{\alpha\beta} G^c_{\alpha\beta} \;\;, \\
\mathcal{O}_2^{\mu\nu} &= d_{abc}\, G_a^{\mu\alpha} G^b_{\alpha\beta} G_c^{\beta\nu} \;\;,
\end{aligned} \tag{6}
$$

where $G_a^{\mu\nu}$ is the gluon strength field tensor. The effective current in the QED case ($G_a^{\mu\nu} \to F^{\mu\nu}, \alpha_s \to \alpha_{em}, d_{abc} \to 1$) can be easily identified from the lowest order Euler-Heisenberg Lagrangian (see Ref. [5]).

The correlator of the induced current (5) is then evaluated in the configuration space giving :

$$
\langle 0| T\, J_\mu(x)\;\, J_\nu^\dagger(0)\, |0\rangle \;=\; -\frac{34}{2025\pi^4 M^8}\left(\frac{\alpha_s}{\pi}\right)^3 d_{abc}d_{abc}\,\left(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2\right)\frac{1}{x^{12}}\,. \tag{7}
$$

In momentum space we need to perform the Fourier transform of Eq. (7). Following the differential regularization procedure [6], which works directly in configuration space, the result for the vacuum polarization contribution of the diagram in Fig. 1(b) at small q^2 reads

$$
\Pi_{\mu\nu}(q) \;=\; \frac{17}{2916000\pi^2}\, d_{abc}d_{abc}\left(\frac{\alpha_s}{\pi}\right)^3 (q_\mu q_\nu - q^2 g_{\mu\nu})\left(\frac{q^2}{4M^2}\right)^4 \ln\left(\frac{\mu^2}{-q^2}\right) + \mathcal{O}\Big[\Big(\frac{q^2}{M^2}\Big)^5\Big]\,, \tag{8}
$$

with μ the renormalization point in this scheme, and $d_{abc}d_{abc} = 40/3$.

As noticed by Groote and Pivovarov [1], moments associated to the diagram in Fig. 1(b) are not defined if $n \geq 4$. Indeed differentiating Eq. (8) four times, at $q^2 \approx 0$, we get:

$$
\frac{1}{4!}\left(\frac{d}{dq^2}\right)^4 \Pi(q^2)|_{q^2\approx 0} = \frac{17}{218700\pi^2}\left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{1}{4M^2}\right)^4\left[\ln\left(\frac{\mu^2}{-q^2}\right) - \frac{25}{12}\right] + \mathcal{O}\Big[\frac{q^2}{M^{10}}\Big] \;\;, \tag{9}
$$

whose real part clearly diverges if we set $q^2 = 0$. Larger n moments are also infrared divergent, and so the authors of Ref. [1] conclude that the standard sum rule analysis must limit the accuracy of theoretical calculations for the $n \geq 4$ moments to the $\mathcal{O}(\alpha_s^2)$ order of perturbation theory.

One obvious way out of this infrared problem is to avoid the $q^2 = 0$ point. As it has been discussed in Ref. [5], this solution is rather ill–conditioned from the phenomenological side though. Moreover we notice (as commented earlier) that it is not possible to implement, from the available experimental information, the second term in the right–hand side of Eq. (4). However, if one does not insist in using full vacuum polarization for the sum rule analysis there is a way to overcome this infrared problem.

3 Infrared safe definition of the moments

The study of analytic properties of perturbation theory amplitudes shows that their singularities are isolated and, therefore, we can discuss each singularity of a perturbative amplitude by itself [7]. As a consequence, $\Pi(q^2)$ in Eq. (1) satisfies a dispersion relation from Cauchy's theorem [3] :

$$\widehat{\Pi}(q^2) \, = \, \sum_n \frac{q^2}{\pi} \int_{s_n}^{\infty} \frac{ds}{s} \, \frac{[\,\Pi(s)\,]_n}{s \, - \, q^2 \, - \, i\epsilon} \quad . \tag{10}$$

Here $[\Pi(s)]_n$ provides the discontinuity across a branch cut starting at the branch point s_n.

In the perturbative calculation, every discontinuity function $[\Pi(s)]_n$ can be associated to a "reduced" Feynman diagram obtained by contracting internal off–shell propagators to a point and leaving internal on–shell lines untouched. Its contribution is written down following the Cutkosky rules for the graph. The reduced diagram corresponding to the massless cut in Fig. 1(a) has the topological structure of the part (b) of that Figure. Let us emphasize though that our following discussion is not grounded on the $q^2 \ll M^2$ regime where the fermion loops have been integrated out : the reduced diagram is just a symbol that specifies a singularity, and the black dots in Fig. 1(b) keep all the analytical structure of the fermion loops.

In a general diagram the discontinuity across a specified cut needs not to be a pure real function in the physical region, only the sum of all cuts in a given channel gives the total imaginary part. Hence the separation between the imaginary parts coming from different final states, as performed in Eq. (4) for the vacuum polarization, does not seem to come directly from the Cutkosky rules. Nevertheless in the heavy quark correlator the discontinuity across the three–gluon cut gives a contribution to the spectral function that is unequivocally real :

$$[\,\Pi(s)\,]_{3g} \, = \, \mathrm{Im}\,\Pi_{3g}(s) \, = \, - \frac{1}{6s} \int dR_{3g} \, \langle \, 0 \, | \, j^{\mu} \, | \, 3\,g \, \rangle \, \langle \, 3\,g \, | \, j_{\mu}^{\dagger} \, | \, 0 \, \rangle \; , \tag{11}$$

from which the dispersive part can be evaluated independently of the $Q\overline{Q}$ cuts [4]. Accordingly we conclude that we can identify and isolate the troublesome massless cut contribution to the two–point function. Indeed Eqs. (10) and (11) justify our previous Eq. (4). This assertion might seem obvious but it is not : A $Q\overline{Q}$ cut on the right–hand fermion loop in Fig. 1(a) does not provide, by itself, a pure real contribution. Only when both $Q\overline{Q}$ cuts, on the left–hand and right–hand fermion loops of Fig. 1(a), are added we get $\mathrm{Im}\Pi_{Q\overline{Q}}$.

Let us go back then to Eq. (4). All the difficulty with the phenomenological application of the sum rules is now the fact that the contribution from the three–gluon cut is contained in both sides of the equality. Thus we propose an *infrared safe* definition of the moments by the trivial subtraction :

$$\widehat{\Pi}_{Q\overline{Q}}(q^2) \;\; \doteq \;\; \widehat{\Pi}(q^2) - \frac{q^2}{\pi} \int_0^{\infty} \frac{ds}{s} \, \frac{\mathrm{Im}\,\Pi_{3g}(s)}{s - q^2 - i\epsilon} \;\; = \;\; \frac{q^2}{\pi} \int_{4M^2}^{\infty} \frac{ds}{s} \, \frac{\mathrm{Im}\,\Pi_{Q\overline{Q}}(s)}{s - q^2 - i\epsilon} \;\; , \tag{12}$$

$$\widetilde{\mathcal{M}}_n \;\; \doteq \;\; \mathcal{M}_n - \frac{1}{\pi} \int_0^{\infty} ds \, \frac{\mathrm{Im}\,\Pi_{3g}(s)}{s^{n+1}} \quad . \tag{13}$$

[3]This expression also gives the residue R_i of a pole at $s = s_i$ if we interpret the discontinuity as $\mathrm{Im}\Pi_i = \pi R_i \delta(s - s_i)$. However we do not consider the existence of $Q\overline{Q}$ Coulomb bound states, as it is not relevant for our discussion.

[4]The integration in Eq. (11) extends to the available three–gluon phase space.

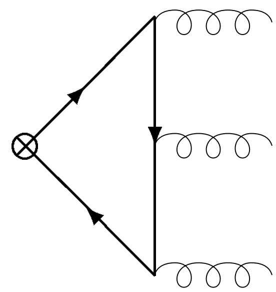


Figure 2: *Feynman diagram for the production of three gluons at $\mathcal{O}(\alpha_s^3)$.*

Of course Eqs. (12) and (13) are meaningless unless we give a precise prescription about how to subtract the contribution of the massless cuts represented by $\mathrm{Im}\,\Pi_{3g}$. Our previous discussion gives us the tool to proceed. Once the full $\mathcal{O}(\alpha_s^3)$ $\Pi(s)$ is calculated we can extract the imaginary part starting at $s = 0$ (which should go with a $\theta(s)$ function) for any value of s. It is clear that the $\theta(s)$ and $\theta(s - 4M^2)$ terms in the imaginary part of the vacuum polarization function correspond to three–gluon massless and to $Q\overline{Q}$ cut graphs, respectively, and $\mathrm{Im}\,\Pi_{3g}$ and $\mathrm{Im}\,\Pi_{Q\overline{Q}}$ are easy to distinguish, as Eq. (11) prevents the appearance of mixed $\theta(s) \cdot \theta(s - 4M^2)$ terms. Therefore we identify $\mathrm{Im}\,\Pi_{3g}$ and we now plug it in the dispersion integral of the right–hand side of Eq. (13) and perform such integration. Divergences contained in both this integral and $\mathcal{M}_n$ as $q^2 \to 0$ will cancel with each other if the same infrared regularization is employed in the two quantities. The intuitive choice would be a low-energy cutoff $s_0 > 0$, and Eq. (13) would be more precisely written as:

$$\widetilde{\mathcal{M}}_n \;\equiv\; \lim_{s_0 \to 0^+} \left[\frac{1}{n!} \left(\frac{d}{dq^2} \right)^n \Pi(q^2)|_{q^2=-s_0} \;-\; \frac{1}{\pi} \int_0^\infty \frac{ds}{s} \, \frac{\mathrm{Im}\,\Pi_{3g}(s)}{(s+s_0)^n} \right] \;, \tag{14}$$

where a vanishing therm in the $s_0 \to 0^+$ limit has been omitted.

The evaluation of the $\mathcal{M}_n$ moments at $q^2 = 0 < 4M^2$ made sense because, up to $\mathcal{O}(\alpha_s^2)$, this point is unphysical and the moments are well defined through an analytic continuation from the high–energy region. However note that the absorptive three–gluon contribution starts at $q^2 = 0$ where perturbative QCD is unreliable. This introduces a further new difficulty in evaluating $\mathcal{M}_n$ moments at $q^2 = 0$, as we reach the physical non–perturbative region. Our definition of the moments, $\widetilde{\mathcal{M}}_n$ in Eq. (13), skips this problem by fully eliminating the massless terms and, therefore, the final heavy quark sum rule will only involve physics at $q^2 > 4M^2$.

The general rule given above is valid for all orders of perturbation theory, but it strongly relies in our ability to extract the massless absorptive part from the full result of $\Pi(q^2)$ calculated at a definite order. Beyond $\mathcal{O}(\alpha_s^2)$ complete analytical results for the heavy quark

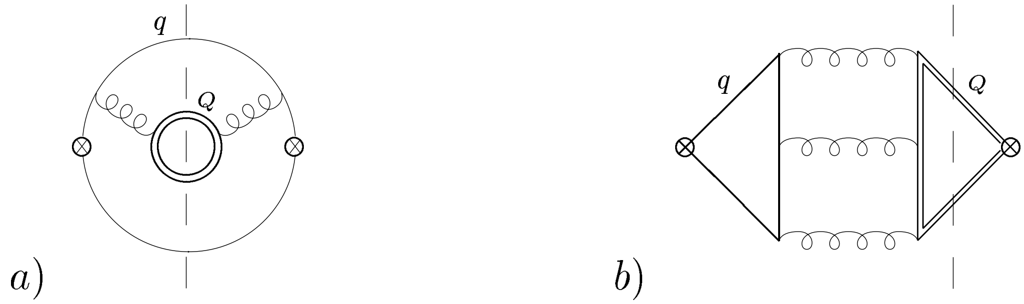


Figure 3: *Examples of perturbative non–heavy quark current correlators at $\mathcal{O}(\alpha_s^2)$ (a) and $\mathcal{O}(\alpha_s^3)$ (b) that contribute to the production of $Q\overline{Q}$ states.*

correlator would be cumbersome and only numerical approaches may be at hand. In this sense, it would be convenient to have a method to calculate $\mathrm{Im}\,\Pi_{Q\overline{Q}}$ only based on Feynman graphs. We have already sketched such a method in the discussion following Eq. (10) : we just need to sum up all the massless cut graphs to get $\mathrm{Im}\,\Pi_{3g}$, and then proceed with the dispersion integration that gives the associated dispersive part [8]. For example, at $\mathcal{O}(\alpha_s^3)$, the only massless absorptive part comes from the three–gluon cut in the diagram of Fig. 1(a); let us call $\mathcal{M}^{\mu}_{3g}$ the amplitude producing three gluons from the heavy quark current at lowest order (i.e. through the quark triangle loop in Fig. 2). The massless contribution to the absorptive part of the correlator is then:

$$\mathrm{Im}\,\Pi_{3g}(s) = -\frac{1}{6s}\int dR_{3g}\;\mathcal{M}^{\mu}_{3g}\cdot\mathcal{M}^{*}_{3g\,\mu} \;\;, \tag{15}$$

with the three–gluon phase space integral defined as

$$\int dR_{3g} \equiv \frac{1}{3!}\frac{1}{(2\pi)^5}\frac{\pi^2}{4s}\int_0^s ds_1\int_0^{s-s_1} ds_2 \;\;, \tag{16}$$

in terms of the invariants $s_1 \equiv (k_1+k_2)^2 = (q-k_3)^2$ and $s_2 \equiv (k_2+k_3)^2 = (q-k_1)^2$, and k_i being the momenta of the gluons. The real part would be obtained by integrating Eq. (15) :

$$\frac{s_0}{\pi}\int_0^\infty \frac{ds}{s}\,\frac{\mathrm{Im}\,\Pi_{3g}(s)}{s+s_0} = \frac{-s_0}{288(2\pi)^4}\int_0^\infty \frac{ds}{s^3(s+s_0)}\int_0^s ds_1\int_0^{s-s_1} ds_2\;\mathcal{M}^{\mu}_{3g}\cdot\mathcal{M}^{*}_{3g\,\mu}\,, \tag{17}$$

which, in principle, could be performed also numerically. The nth-derivative of relation (17) respect to s_0, in the limit $s_0 \to 0^+$, would give the infrared divergent contribution that should be subtracted from the full moments, as dictated by Eq. (14).

The method discussed in this Section could be extended to more general two–point correlators involving intermediate $Q\overline{Q}$ states in their perturbative expansion. Indeed the correlator of two light quark vector currents has $\mathcal{O}(\alpha_s^2)$ contributions with an internal loop of heavy quarks (Fig. 3 (a)) and, similarly, the asymmetric correlator of a heavy and a light vector quark currents is no longer vanishing at $\mathcal{O}(\alpha_s^3)$ (Fig. 3 (b)). The absorptive part coming from the $Q\overline{Q}$ cuts in the previous examples contribute to the phenomenological input $\sigma(e^+e^- \rightarrow Q\overline{Q})$ in the usual sum rule analysis. Correspondingly they should be accounted for in the theoretical side. In short, the production of $Q\overline{Q}$ states concerns not only the correlator of a couple of heavy quark currents and, for a more rigorous use of the sum rules method, this imbalance should be taken into account and properly fixed. A two–point function built from the sum of the electromagnetic currents associated to each quark flavour could be used in a generalized version of Eq. (1) :

$$\Pi^G_{\mu\nu}(p) \; = \; i \int d^4x \, e^{ipx} \sum_{q,q'} \langle \, 0 \, | \, T \, \left(\, \overline{q}(x) \, \gamma_\mu \, q(x) \, \right) \, \left(\, \overline{q'}(0) \, \gamma_\nu \, q'(0) \, \right) | \, 0 \, \rangle \quad , \tag{18}$$

where now q and q' stand for heavy or light quarks indistinctly, and $q = q'$ is also allowed. As the different absorptive cuts contribute additively to $\mathrm{Im}\Pi(p^2)$, the unwanted light quark and gluon $q\overline{q}$, $q\overline{q}g$, ggg, $\ldots$ cuts could be identified and, through the dispersive technique, subtracted from the full $\Pi(p^2)$ result. Consequently we are left with every possible $Q\overline{Q}$ intermediate state arising from vector current production. Notwithstanding, the feasibility of this procedure from a technical point of view appears rather cumbersome and, at present, the experimental accuracy in the measurement of $\sigma(e^+e^- \rightarrow Q\overline{Q})$ cannot accommodate the corrections just discussed.

4 Conclusions

We have shown that rigorous and straightforward results of the general theory of singularities of perturbation theory amplitudes provide all–important tools to extract the unwanted $\mathcal{O}(\alpha_s^3)$ three–gluon massless cut pointed out by Groote and Pivovarov from the vector current correlator of heavy quarks. We conclude that the appropriate procedure to obtain information about the heavy quark parameters should make use of the infrared safe corrected moments, defined in Eq. (14), that now indeed satisfy the modified sum rule :

$$\widetilde{\mathcal{M}}_n \; = \; \frac{1}{\pi} \int_{4M^2}^{\infty} ds \, \frac{\mathrm{Im}\,\Pi_{Q\overline{Q}}(s)}{s^{n+1}} \quad , \tag{19}$$

where the right–hand side can be extracted from the heavy quark production cross section $\sigma(e^+e^- \rightarrow Q\overline{Q})$.

Finally we have pointed out that, starting already at $\mathcal{O}(\alpha_s^2)$, the use of sum rules associated to heavy vector current correlators shows and imbalance between the phenomenological input in the dispersion relation and the perturbative two–point function. This is due to the fact that the cross section of production of $Q\overline{Q}$ heavy quarks is contained not only in the correlator of two heavy quark currents but in those involving light quarks too. We have indicated how to improve the application of sum rules by constructing a generalized correlator of vector currents in Eq. (18) and, afterwards, extracting all the perturbative information not related with the production of heavy quarks, as we did in detail for the three–gluon cut.

Acknowledgements

We wish to thank A. Pich for calling our attention on this problem. We also thank G. Amorós, M. Eidemüller and A. Pich for relevant discussions on the topic of this paper and for reading the manuscript. The work of P. D. Ruiz-Femenía has been partially supported by a FPU scholarship of the Spanish *Ministerio de Educación y Cultura*. This work has been supported in part by TMR, EC Contract No. ERB FMRX-CT98-0169 and by CICYT (Spain) under grant PB97–1261.

References

[1] S. Groote and A. A. Pivovarov, "Low-energy gluon contributions to the vacuum polarization of heavy quarks", [hep-ph/0103047].

[2] T. Appelquist and H. Georgi, Phys. Rev. **D8** (1973) 4000;
A. Zee, Phys. Rev. **D8** (1973) 4038.

[3] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385;
M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 448;
L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1;
M. Jamin and A. Pich, Nucl. Phys. B **507** (1997) 334 [hep-ph/9702276];
A. H. Hoang, Phys. Rev. D **59** (1999) 014039 [hep-ph/9803454];
M. Beneke and A. Signer, Phys. Lett. B **471** (1999) 233 [hep-ph/9906475];
M. Eidemuller and M. Jamin, Phys. Lett. B **498** (2001) 203 [hep-ph/0010334].

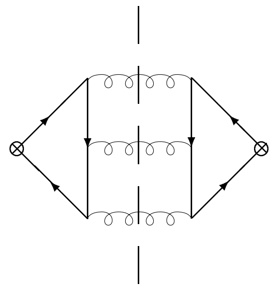
[4] K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, Nucl. Phys. B **482** (1996) 213 [hep-ph/9606230];
K. G. Chetyrkin, R. Harlander, J. H. Kuhn and M. Steinhauser, Nucl. Phys. B **503** (1997) 339 [hep-ph/9704222].

[5] S. Groote and A. A. Pivovarov, "Heavy quark induced effective action for gauge fields in the SU(N(c)) x U(1) model and the low-energy structure of heavy quark current correlators", [hep-ph/0103313].

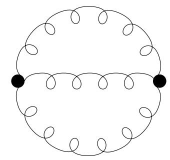
[6] D. Z. Freedman, K. Johnson and J. I. Latorre, Nucl. Phys. B **371** (1992) 353;
D. Z. Freedman, G. Grignani, K. Johnson and N. Rius, Annals Phys. **218** (1992) 75 [hep-th/9204004].

[7] L. D. Landau, Nucl. Phys. **13** (1959) 181;
J. C. Taylor, Phys. Rev. **117** (1960) 261;
R. E. Cutkosky, J. Math. Phys. **1** (1960) 429;
R. E. Cutkosky, Rev. Mod. Phys. **33** (1961) 448.

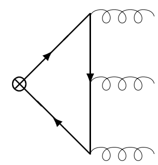
[8] J. Portolés and P. D. Ruiz–Femenía, work in progress.

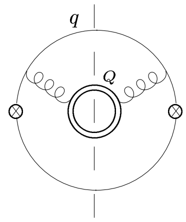


$a)$

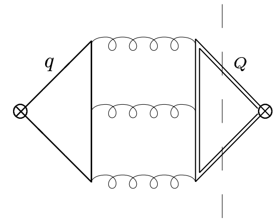


$b)$





$a)$



$b)$